Swing Models

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- How many seats will they win in parliament? District-level polls are expensive, so we usually impute using national-level polls.
- For decades, two competing estimation methods have been used.
 - Uniform: the additive national swing is the same in all districts.
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District	National	1	2	3	4	5
Election 2016	0.40	0.40	0.32	0.48	0.10	0.70
Election 2020 polling	0.60	?	?	?	?	?
Uniform	0.60	0.60	0.52	0.68		
Proportional	0.60	0.60	0.48	0.72	0.15	1.05

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Uniform and proportional fail some natural axioms.

- ▶ We find a simple model that does satisfy all axioms:
 - defined piecewise for positive and negative swings;
 - of proportional type;
 - considers nonvoters rather than voters.
- Dataset of tens of thousands of US Congressional elections: piecewise model consistently performs a little better than others.
- Conclusion: throw out your old swing model, and replace with the new one. And keep looking for better models, because none of them are great in practice!
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► Assume *K* districts of equal size and two parties, *A* and *B*, contesting all districts.

- Unless otherwise specified we state results for party A, whose vote share is denoted x_i.
- The aggregate vote share is denoted \overline{x} .
- ► We consider two elections: one for which we know the results and another for which we don't. For the latter, we use prime for everything: x'_i, x'.
- In addition to election prediction, this comes up in the study of gerrymandering and electoral system design, where we need to discuss counterfactuals.

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Definition

The district-level swing in district i is given by

$$s_i := x_i' - x_i.$$

The aggregate swing is given by

$$s := \overline{x}' - \overline{x}.$$

By a naive swing model we mean a prediction of x' of the form $s_i = f(x_i, s)$ where $f \equiv f_A$ is a fixed function (depending only on A but not i or s).

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• Example: uniform swing, for which $s_i = s$.

Desirable properties of a swing model

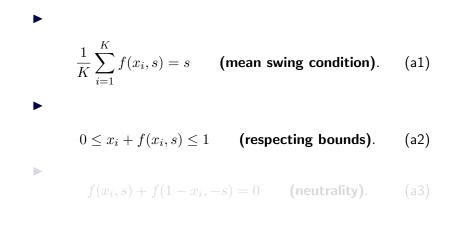
$$\frac{1}{K}\sum_{i=1}^{K} f(x_i, s) = s \quad \text{(mean swing condition)}. \quad (a1)$$

$$0 \le x_i + f(x_i, s) \le 1 \quad \text{(respecting bounds)}. \quad (a2)$$

$$f(x_i, s) + f(1 - x_i, -s) = 0 \quad \text{(neutrality)}. \quad (a3)$$

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No naive swing model can satisfy both (a1) and (a2).

► We need a larger family, so we consider swing models of the form s_i = f(x_i, s, x̄).

SQA

Example: proportional swing, for which $s_i = sx_i/\overline{x}$.

► However:

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Proposition

A new model

Define

$$f(x_i, s, \overline{x}) = \begin{cases} s \frac{1-x_i}{1-\overline{x}} & \text{if } s \ge 0; \\ s \frac{x_i}{\overline{x}} & \text{if } s < 0. \end{cases}$$

- This looks like proportional swing but differs by considering positive and negative swings differently.
- Justification:
 - suppose in each district there are swing voters as well as partisans;
 - in districts where A already scores highly, there are relatively few swing voters left to convince;
 - in districts where A scored relatively low, there is more chance of winning over swing voters;
 - if the swing is away from A, the reverse is true (alternatively, the same is true of B).

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Table: Axioms satisfied by swing model

Model/Axiom	a1	a2	a3
uniform	1	X	1
proportional	\checkmark	X	X
truncated uniform	X	1	\checkmark
linear in s	\checkmark	X	(✔)
piecewise	\checkmark	\checkmark	\checkmark

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- Proportional: larger changes in already strong districts.
- Piecewise: smaller changes in already strong districts.

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Table: Predictions for "polarized" example with 2 parties and 2 districts, swing of 2α to A

model / (party, district)	(A,1)	(A,2)
original	$1 - \alpha$	α
uniform	$1 + \alpha$	3lpha
proportional	$1+3\alpha-4\alpha^2$	$\alpha + 4\alpha^2$
piecewise	$1 - \alpha + 4\alpha^2$	$5\alpha - 4\alpha^2$

So for big swings and polarized districts, there is a big difference in predictions.

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Table: Predictions for "competitive" example with 2 parties and 2 districts, swing of 2ε to A

	(A,1)	(A,2)
original	$1/2 - \varepsilon$	$1/2 + \varepsilon$
uniform	$1/2 + \varepsilon$	$1/2 + 3\varepsilon$
proportional	$1/2 + \varepsilon - 4\varepsilon^2$	$1/2 + 3\varepsilon + 4\varepsilon^2$
piecewise	$1/2 + \varepsilon + 4\varepsilon^2$	$1/2 + 3\varepsilon - 4\varepsilon^2$

So it is not surprising that in many real elections, all methods have fairly similar performance.

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Table: Results for swing models on standard dataset. Bold entries indicate the best performance among the models on the given measure corresponding to the column for the given dataset.

dataset	model / measure	winner	sign	bounds	mean-square
unc0.75	uniform	0.932	0.497	1.000	0.00747
unc0.75	proportional	0.933	0.497	0.999	0.00756
unc0.75	piecewise	0.930	0.497	1.000	0.00728
unc1.0	uniform	0.904	0.498	0.832	0.0381
unc1.0	proportional	0.904	0.539	0.884	0.0389
unc1.0	piecewise	0.892	0.604	1.000	0.0360
cont only	uniform	0.855	0.678	1.000	0.00521
cont only	proportional	0.853	0.678	0.999	0.00533
cont only	piecewise	0.852	0.678	1.000	0.00509

The piecewise model is better justified than the others, and should henceforth be adopted universally.

- For most applications, it may not make a difference which swing method we use, if we care only about number of seats won.
- No existing method predicts close elections or even the sign of the district-level swing well, so more work is desirable.
- There may be other non-electoral applications of swing models, where we care more about the fine details.
- To my knowledge, machine learning methods have not been widely used to predict swing, but perhaps should be investigated.

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- The piecewise model is better justified than the others, and should henceforth be adopted universally.
- For most applications, it may not make a difference which swing method we use, if we care only about number of seats won.
- No existing method predicts close elections or even the sign of the district-level swing well, so more work is desirable.
- There may be other non-electoral applications of swing models, where we care more about the fine details.
- To my knowledge, machine learning methods have not been widely used to predict swing, but perhaps should be investigated.