Where are the best parliamentary election methods?

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- Looking to build links with UC Irvine.
- Recently awarded grant to study multi-winner elections (with A. Slinko and G. Pritchard). Part of this involves a systematic study of tradeoffs.

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- It is common for a government to be formed by some subset of the parties in the assembly.
- Two key issues are representation (how well the elected representatives reflect the views of the voters) and stability of government. It is far from clear how to measure these. Each voter may have a single identified representative, or may not.
- It is widely accepted that there is an inevitable tradeoff between representation and stability. At the extremes (elected dictator versus direct democracy) this seems clear enough.

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- It is possible *a priori* that the two criteria can be co-optimized only at the extremes, or somewhere in between the extremes.
- In any case, we seek Pareto optimal values, not clearly dominated in both dimensions.
- From the perspective of designing a mechanism, we must consider many (all?) possible distributions of votes.

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- They studied 609 election outcomes from 81 countries during 1945–2006.
- They try to control for some electoral system factors, such as thresholds, and many socioeconomic factors.
- They conclude that low to moderate (say 3–7) "district magnitude" is well correlated with the best tradeoff. *Some countries — such as Costa Rica, Hungary, Ireland, Portugal, and Spain — appear to have discovered a "sweet spot" in the design of electoral systems.*

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- How is proportionality measured?
- How is fragmentation measured?
- There are apparent errors in the data.

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- Even in the above families, there are many parameters (district magnitude, thresholds, ...) to be optimized.

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- Eggers & Fouirnaies (2014) show that for plurality-based systems, district magnitude may not relate to proportionality as Carey & Hix claim.

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- Disproportionality measures are party-based. There are other misrepresentation measures for systems without parties.

Possible improvements - disproportionality

- Koppel & Diskin (2009) give 8 axioms for a disproportionality measure and show that the cosine measure

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- Wada (2010, 2012) presents generalised entropy measures for disproportionality, with explicit social welfare underpinnings.
- What does "disproportionality" mean without plurality?

Improvements - fragmentation

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- The same idea for Banzhaf's index has been suggested by Caulier & Dumont (preprint). However this makes sense only for "take it or leave it" committees (Laruelle & Valenciano) rather than "bargaining committees".
- We can replace s_i by the Shapley-Shubik power index σ_i. This has a noncooperative bargaining interpretation (L & V, 2007).

Outline of analysis

- We simulate many artificial societies (preference distributions on voters). We assume independent districts (for now), and use several probability models, e.g. Polyà-Eggenberger, 2D spatial, perturbations of real data.

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- We use the (competing) measures of system performance above.
- We aim to distinguish between competitive and clearly Pareto-suboptimal parameter settings.
- We (Fowlie & Wilson 2012) have done this on a much smaller scale in the context of a review of the NZ voting system.

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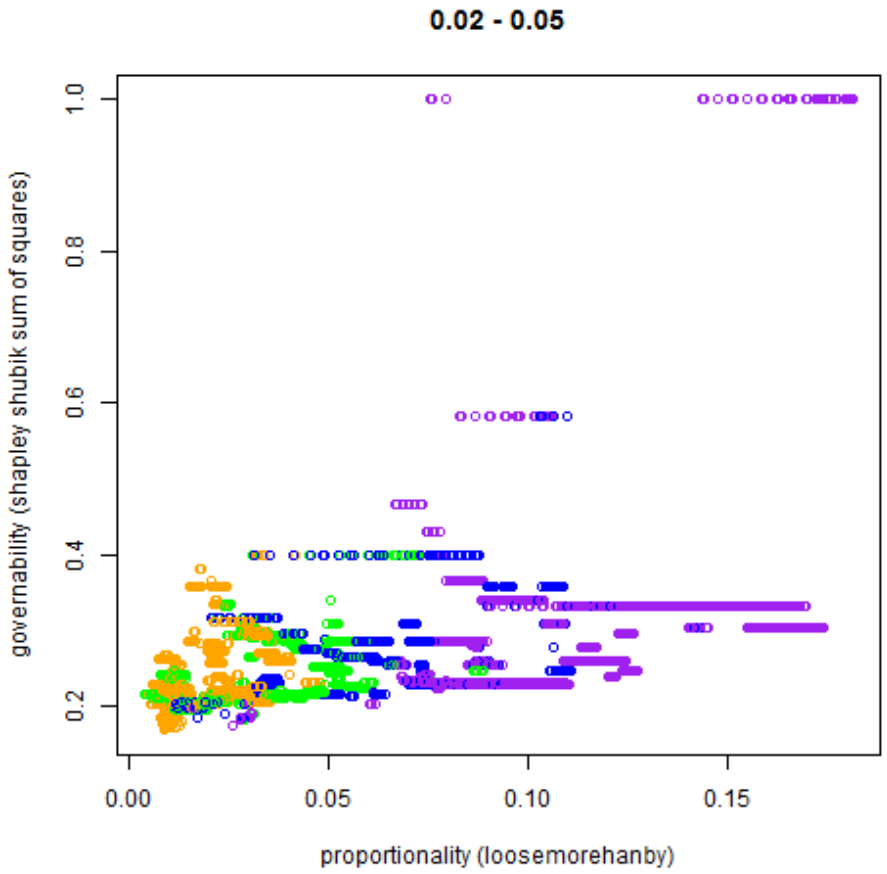
- We assume no difference in strategic voter behaviour, or party behaviour.

Example: NZ system, Loosemore-Hanby/Shapley-Shubik

Figure: threshold 2% (orange), 3% (green), 4% (blue), 5% (purple)



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- How to compare simulation results for different parameters? When does one parameter value dominate another in the approximate Pareto sense?
- Which formal measures of robustness of results should we use?
- Are we measuring the right things?