# Electoral engineering via simulation 

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UC Davis, 2013-05-22

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- A submission to the NZ Electoral Commission on proposed changes to the current MMP electoral system.
- A response (in progress) to the paper "The Electoral Sweet Spot", by John M. Carey and Simon Hix, American Journal of Political Science, 2011.


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- Electoral Commission materials contained much information about the various systems, much of it purely qualitative.
- Having been unsatisfied by the level of public debate over the Alternative Vote referendum in UK, we wanted to show voters the likely consequences of changing to these systems.


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- FPP: voter casts vote for single party in district; single representative elected.
- PV: voter casts preferential vote in a district; preferential vote in a district. Basically, STV with a single winner in each district.
- SM: hybrid of FPP and proportional system; some representatives elected as in FPP, others proportionally.


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- No changes to some special (Maori or South Island) district seat distribution principles.
- SM will have 90 district and 30 list seats.
- (not a hard constraint) Under STV, "It is likely the 120 MPs would be divided between 24 and 30 districts, each with 3 to 7 MPs."


## Basic features of the simulator

- User enters percentage national support levels for each party. The list of parties is pre-selected, and additions and deletions can be done by the user.


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- Calculator outputs seat distributions in a 120 -seat Parliament for each party under each of the 5 systems.
- For MMP, user can also adjust some parameters: the threshold (default 5\%) and whether threshold is waived for district winners (default yes).
- Important: no district- or candidate-specific information is part of the input. There are good reasons for this, in terms of complexity.


## Challenge: voter behaviour?

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- Our simulator avoids this part by leaving most of it up to the user.


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- Instead, we need to downscale the overall party support level to district-level results.
- The way this done affects the result for all the systems except MMP.


## Downscaling party support

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- We assume each party's support is distributed among districts in the same manner as the 2008 party vote. That is, any variations in support apply uniformly across districts.
- There is some evidence that this is reasonable, at least for MMP elections:

| National Party | 1999 | 2008 |
| :--- | :---: | :---: |
| Total party vote | 629932 | 1053398 |
| comprising: |  |  |
| Mangere | $0.45 \%$ | $0.39 \%$ |
| Mt. Albert | $1.08 \%$ | $1.18 \%$ |
| Nelson | $1.50 \%$ | $1.46 \%$ |
| Clutha-Southland | $2.04 \%$ | $1.92 \%$ |

## Testing the downscaling method

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- Although the ("third") parties, districts and society have changed since then, simulated results are remarkably close to the actual ones. Note that our algorithm uses only the 2008 party vote.
- Downscaling is widely used for election forecasting in FPP elections. We also compared our results to the real seat allocations in historical UK and Canadian elections, and they were reasonably close.


## Testing the downscaling method

- We used input from historical NZ elections under FPP, from 1935 to 1993.
- Although the ("third") parties, districts and society have changed since then, simulated results are remarkably close to the actual ones. Note that our algorithm uses only the 2008 party vote.
- Downscaling is widely used for election forecasting in FPP elections. We also compared our results to the real seat allocations in historical UK and Canadian elections, and they were reasonably close.
- Is there a standard way to test quality of performance of such an algorithm?

| Year | NAT sim | LAB sim | others sim | NAT | LAB | others |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1993 | 54 | 45 | 0 | 50 | 45 | 4 |
| 1990 | 71 | 26 | 0 | 67 | 29 | 1 |
| 1987 | 50 | 47 | 0 | 57 | 40 | 0 |
| 1984 | 37 | 58 | 0 | 37 | 56 | 2 |
| 1981 | 48 | 44 | 0 | 47 | 43 | 2 |
| 1978 | 48 | 44 | 0 | 51 | 40 | 1 |
| 1975 | 55 | 32 | 0 | 55 | 32 | 0 |
| 1972 | 33 | 54 | 0 | 32 | 55 | 0 |
| 1969 | 47 | 37 | 0 | 45 | 39 | 0 |
| 1966 | 47 | 33 | 0 | 44 | 35 | 1 |
| 1963 | 47 | 33 | 0 | 45 | 35 | 0 |
| 1960 | 48 | 32 | 0 | 46 | 34 | 0 |
| 1957 | 41 | 39 | 0 | 41 | 39 | 0 |
| 1954 | 43 | 37 | 0 | 45 | 35 | 0 |

Table: Real and simulated seat distributions in NZ Parliament

## Challenge: redistricting

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- We need:
- 120 districts for FPP and PV;
- 90 districts for SM;
- 24-30 districts for STV.
- NZ currently has 70 districts: 63 general and 7 Maori.
- We need to be able to disaggregate the 2008 party vote into our new districts, for downscaling purposes.


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- About $20 \%$ of votes are not traceable to a polling place, but only to a district. (Advance votes, special votes, overseas votes, etc.)


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- It's a lot of work! (5604 polling places, counting multiplicities).
- About $20 \%$ of votes are not traceable to a polling place, but only to a district. (Advance votes, special votes, overseas votes, etc.)
- Instead: define each new district as a convex combination of contiguous current districts.

Turn 70 old districts into 120 new ones:


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(The new districts are $7 / 12$ the size of the old ones.)

## Challenge: preference-order votes

- For PV and STV we must infer voter preference orders over candidates, given only the voter's first choice.
- Asking the user to enter preference orders leads to an infeasible burden of data entry.


## Preference orders

- We use the 2008 NZ Election Study (www.nzes.org) data to infer the frequency of preference orders given the first choice.


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- An NZES survey question asks respondents to rate each party from 0 to 10 , with sample size around 3000 . We converted these numerical scores into preference orders, breaking ties uniformly.
- We have separate collections of preference orders for the general and Maori rolls.


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## Challenge: ill-posed problem

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- We made concrete choices based on our judgment and documented them.
- Specifically, we created virtual districts with 3-7 representatives per district. We used Wright's method with above-the-line voting (preference order on parties, not on individual candidates).


## Results: poll 3 months before referendum

One News/Colmar Brunton poll 21/8/2011:

|  | Vote | MMP | FPP | PV | STV | SM |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| National | $56 \%$ | 70 | 101 | 98 | 71 | 95 |
| Labour | $30 \%$ | 37 | 19 | 22 | 45 | 21 |
| Green | $6.0 \%$ | 7 | 0 | 0 | 1 | 2 |
| NZF | $2.3 \%$ | 0 | 0 | 0 | 0 | 1 |
| ACT | $1.7 \%$ | 2 | 0 | 0 | 0 | 1 |
| Maori | $1.4 \%$ | 2 | 0 | 0 | 3 | 0 |
| United | $0.5 \%$ | 1 | 0 | 0 | 0 | 0 |
| Mana | $0.9 \%$ | 1 | 0 | 0 | 0 | 0 |
| Total |  | 120 | 120 | 120 | 120 | 120 |

(MMP district seats: Maori 2; ACT, United, Mana 1 each.)

## Results: 2011 general election party vote

|  | Vote | MMP | FPP | PV | STV | SM |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| National | $47.31 \%$ | 59 | 99 | 86 | 67 | 90 |
| Labour | $27.48 \%$ | 34 | 21 | 34 | 46 | 24 |
| Green | $11.06 \%$ | 14 | 0 | 0 | 4 | 4 |
| NZF | $6.59 \%$ | 8 | 0 | 0 | 0 | 2 |
| ACT | $1.07 \%$ | 1 | 0 | 0 | 0 | 0 |
| Maori | $1.43 \%$ | 3 | 0 | 0 | 3 | 0 |
| United | $0.6 \%$ | 1 | 0 | 0 | 0 | 0 |
| Mana | $1.08 \%$ | 1 | 0 | 0 | 0 | 0 |
| Conservative | $2.65 \%$ | 0 | 0 | 0 | 0 | 1 |
| Total | 99.27 | 122 | 120 | 120 | 120 | 120 |

(MMP district seats: Maori 3; ACT, United, Mana 1 each.)

## NZES hypothetical FPP vote

The 2008 NZES survey also included a question:

Imagine that the 2008 election had been held under the old first past the post system, and you had only ONE vote for one of the candidates who stood in your electorate. Which party's candidate would you have voted for?

## Results: 2008 hypothetical FPP vote

|  | Vote | FPP | SM |
| :--- | ---: | ---: | ---: |
| National | $44.8 \%$ | 76 | 70 |
| Labour | $37.6 \%$ | 37 | 40 |
| Green | $3.5 \%$ | 0 | 1 |
| NZF | $2.5 \%$ | 0 | 1 |
| ACT | $0.7 \%$ | 0 | 0 |
| Maori | $5.0 \%$ | 7 | 8 |
| United | $0.4 \%$ | 0 | 0 |
| Progressive | $0.6 \%$ | 0 | 0 |
| Total seats |  | 120 | 120 |

(For SM, we use the same vote-shares for both votes.)

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- We reimplemented the simulator from scratch and allowed for more general input.
- We ran this on historical results from UK and Canadian elections under FPP (this just tests the downscaling algorithm). Results were reasonably good, but not as good as the NZ ones.
- We have not yet tested it on very different electoral systems (e.g. STV in Australia).

| Year | CON sim | LAB sim | OTHER sim | CON | LAB | OTHER |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2010 | 291 | 258 | 96 | 306 | 258 | 81 |
| 2005 | 196 | 331 | 118 | 198 | 355 | 93 |
| 2001 | 186 | 391 | 68 | 166 | 413 | 62 |
| 1997 | 186 | 407 | 66 | 165 | 418 | 76 |

Table: Real and simulated seat distributions in UK House of Commons using FPP popular vote as input

| Year | CON sim | LIB sim | OTHER sim | CON I | LIB | OTHER |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2011 | 154 | 95 | 59 | 166 | 103 | 39 |
| 2008 | 144 | 97 | 67 | 143 | 77 | 88 |
| 2006 | 144 | 97 | 67 | 124 | 103 | 81 |
| 2004 | 111 | 128 | 69 | 107 | 135 | 66 |

Table: Real and simulated seat distributions in Canadian House of Commons using FPP popular vote as input

## Lessons

- Interacting with the public is hard work. We documented everything and all source code was available. However, attacking the results of the simulation because they are emotionally unsatisfying seems much easier than criticizing assumptions or implementation.


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## Lessons

- Interacting with the public is hard work. We documented everything and all source code was available. However, attacking the results of the simulation because they are emotionally unsatisfying seems much easier than criticizing assumptions or implementation.
- This work got more publicity, even among academics, than anything else I have been involved in.
- Public outreach work can lead to interesting and mathematically nontrivial research questions.


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- After over 4000 initial submissions, they produced a Proposal Paper that proposed only the following changes: reduce party vote threshold to $4 \%$; remove one district seat threshold; remove the "overhang" seats.
- http://mmpreview.org.nz/ contains much information on the process.


## Our submission

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## Our submission

- We focused on the value of the party vote threshold, assuming the other proposals are adopted.
- We were not convinced by the Commission's arguments for the optimality of this value.
- We aimed to compute measures of overall system quality under various assumptions on voter preferences.
- We investigated values of threshold from 0 to $8 \%$, and interpret our results as showing that $4 \%$ is considerably too high.


## Representation versus decisiveness

- It seems clear that there should be some sort of trade-off between these criteria for an electoral system. At least at the extremes (dictatorship, direct democracy), they seem to be in conflict.


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- It is possible a priori that the two criteria can be co-optimized only at the extremes, or somewhere in between the extremes.
- If we can agree on measures for these desiderata, we can at least compare electoral systems under a given distribution of votes.
- From the perspective of designing a mechanism, we must consider many (all?) possible distributions of votes.
- We do not consider strategic behaviour at all in this analysis.


## Measures of system quality: proportionality

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- We focus on the concepts of proportionality and governability, which are widely believed to be inversely related in some way.
- There are many disproportionality indices in the literature. Many relate strongly to an apportionment method. Each is computed using the vote fraction $v_{i}$ and seat fraction $s_{i}$ awarded to each party, $i$.
- We use the Loosemore-Hanby index (related to Hamilton's method),

$$
L=\frac{1}{2} \sum_{i}\left|v_{i}-s_{i}\right|
$$

and the Gallagher index,

$$
G=\left(\frac{1}{2} \sum_{i}\left(v_{i}-s_{i}\right)^{2}\right)^{1 / 2}
$$

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- The most commonly used measure is the effective number of parties of Laakso and Taagepara,

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- It makes sense to us also to use a modified version that takes into account power, rather than just presence in Parliament. We replace the fraction $s_{i}$ above by the Shapley-Shubik power index $\sigma_{i}$. This index has an interpretation in terms of a noncooperative bargaining model.


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- This is done at the national level, then disaggregated to districts using the same method as in the referendum simulator.
- We assume no difference in strategic voter behaviour, or party behaviour.


## Results: Loosemore-Hanby/Shapley-Shubik

Figure: 2\% (orange), 3\% (green), 4\% (blue), 5\% (purple)


## Results: Gallagher/Shapley-Shubik

Figure: $2 \%$ (orange), $3 \%$ (green), $4 \%$ (blue), $5 \%$ (purple)


Mark C. Wilson

## Results: Gallagher/Laakso-Taagepara

Figure: 3\% (red) versus 4\% (blue)
$0.03-0.04$


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## Research questions

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- Which formal measures of robustness of results should we use, if any?


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- It was the only submission to receive this treatment.
- However, the Commission stuck with their recommendation of a $4 \%$ threshold.
- This is perhaps disappointing, but at least they didn't recommend an increase (a large number of submitters did favour that).


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- They try to control for some electoral system factors, such as thresholds, and many socioeconomic factors.
- Difficulties: what if the district magnitude is not constant? isn't 609 a rather small number?


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- We use the measures of system performance described above, plus some others.
- We aim to distinguish between competitive and clearly Pareto-suboptimal parameter settings.


## Example of results - "NZ-like societies"

## Simulated Elections



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- However, it seems that the Carey-Hix qualitative description is borne out by our simulations. District magnitude 1 appears to be clearly suboptimal, but the optimal size depends on the type of system used.
- What are the "right" statistical methodologies to use here?
- Questions and comments welcome!

