Multi-district preference modelling

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- Obvious choices that ignore the district structure will give 100% of seats to a party with 51% support, or 51% of seats, neither of which is realistic.

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- The PE model has been used to generate artificial data in the single-district case and for special values of K it yields some known analytically nice preference distributions.

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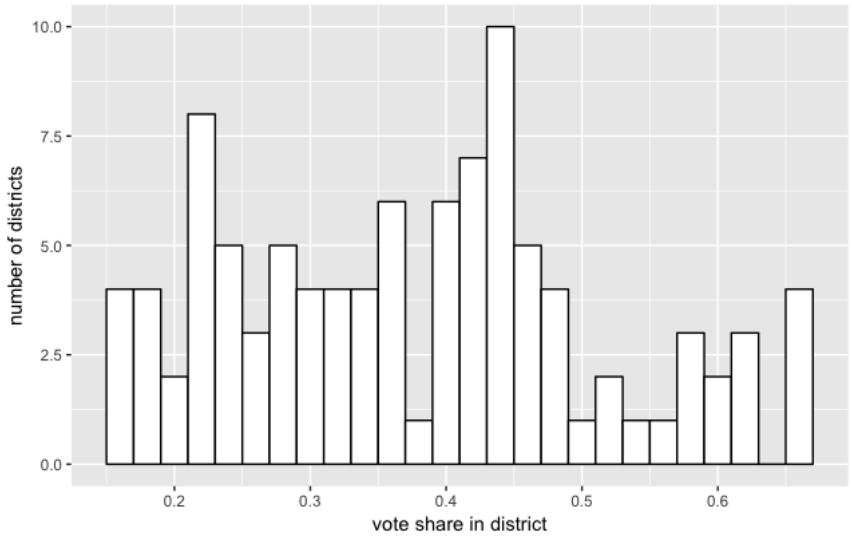
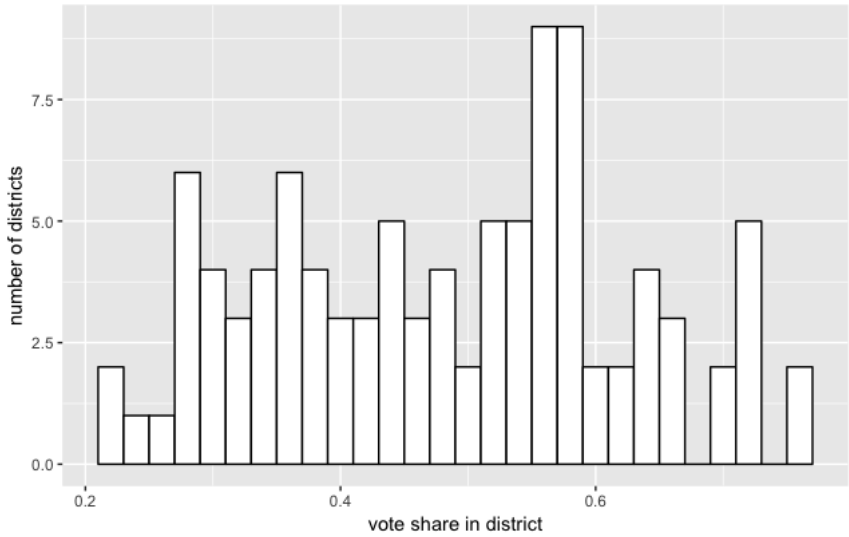
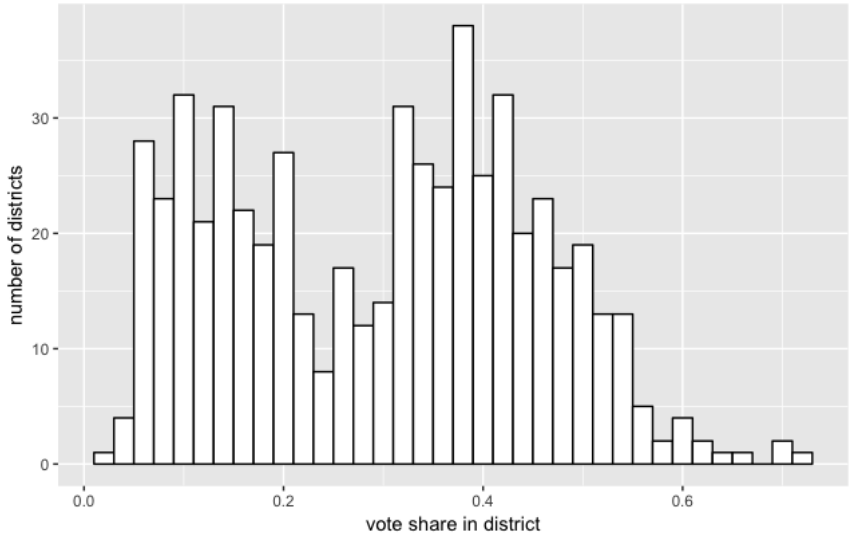
  - choose uniformly a voter from D_2;
  - add K copies of this voter to D_1.

Realistic-looking data is easily obtained ($p = 0.1$)







One of these is UK Labour in 2010, the others are simulations.

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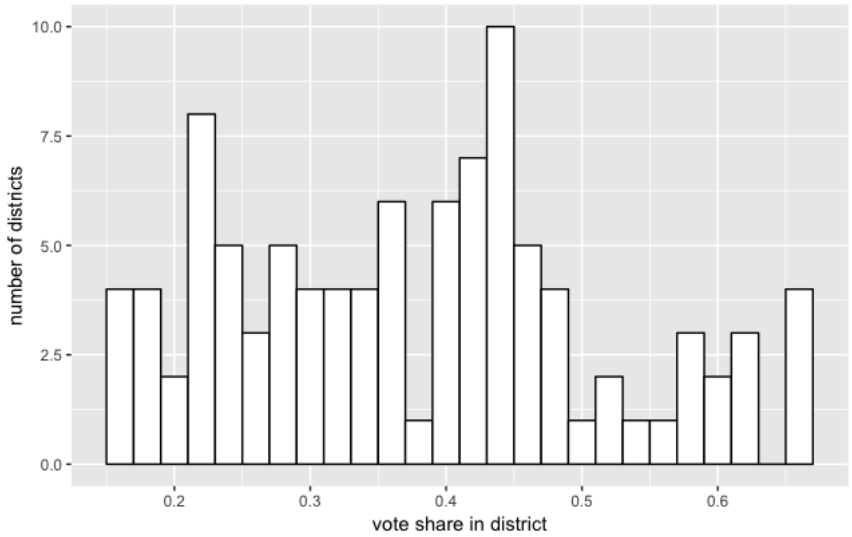

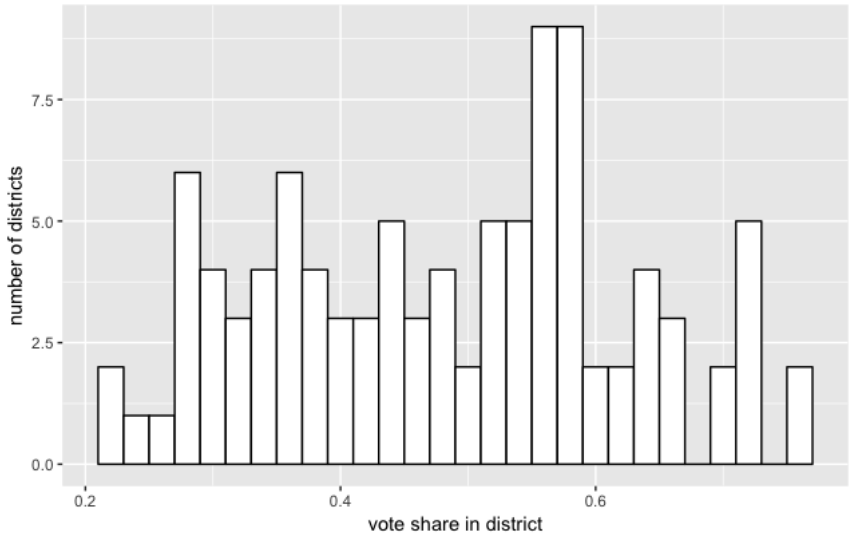

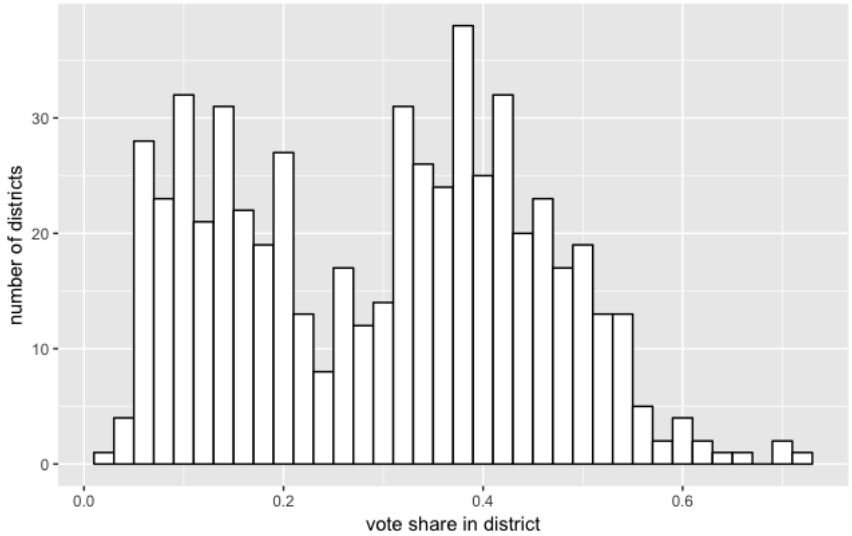

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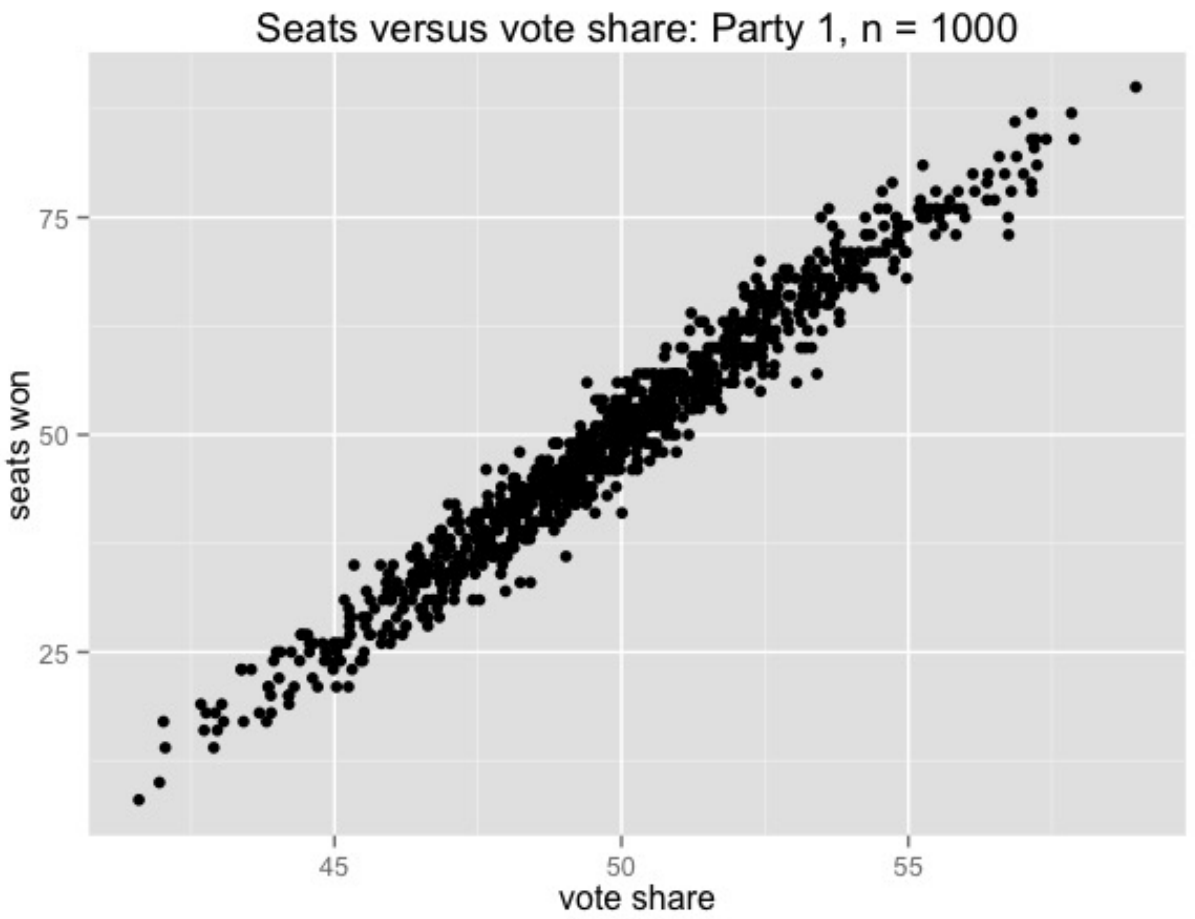
Figure: Vote share vs seat share for party A: $p = 0.2$



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- There are potentially many other applications involving the votes-seats mapping (tradeoff between proportionality and decisiveness, gerrymandering, etc).
- We recommend using this method for all situations where simulated data for such electoral systems is needed.

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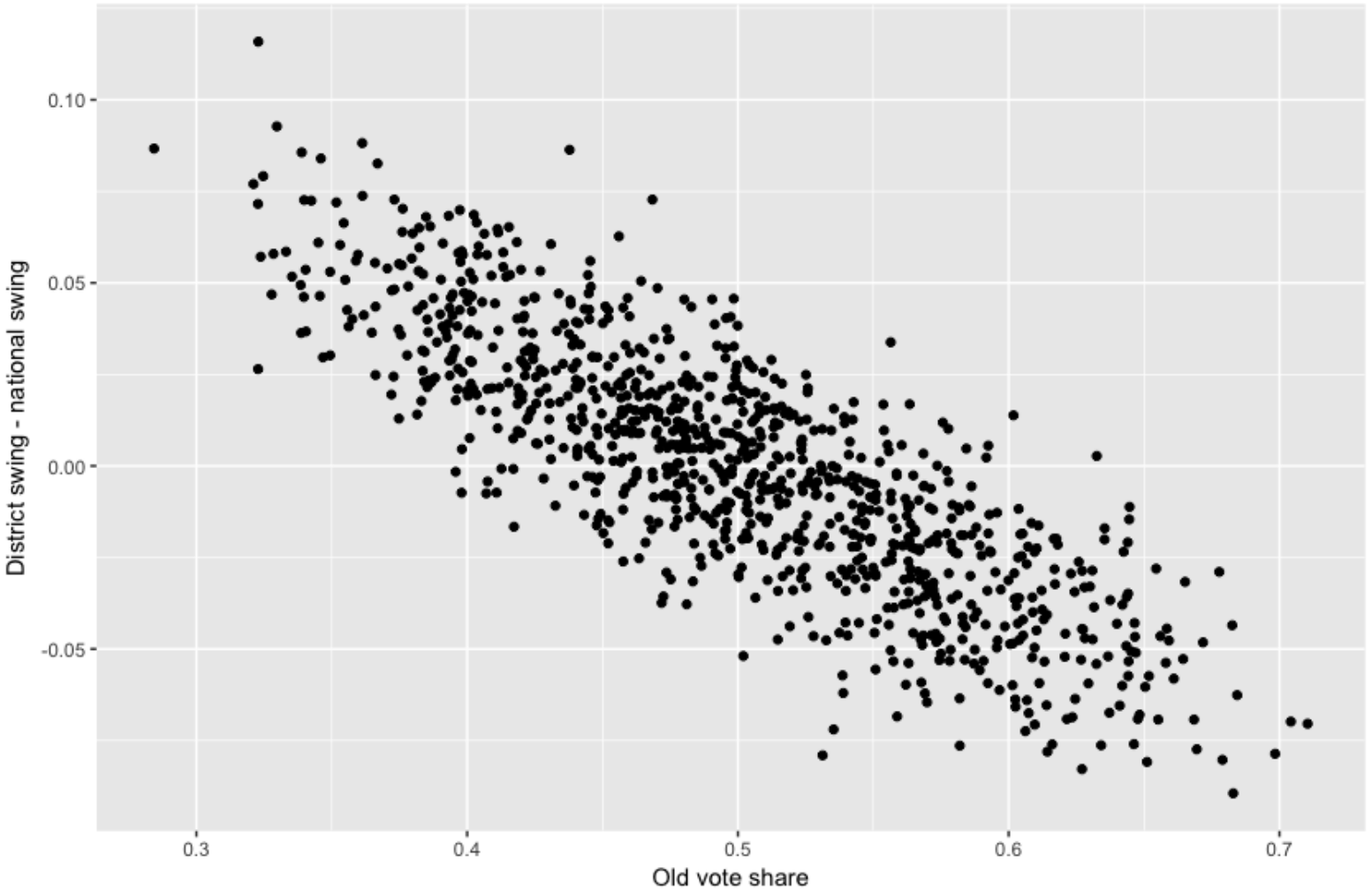
- We can test these out by running our urn process for a while, starting with a fixed election result, to generate a constellation of nearby elections. If our simulated elections are realistic, this should give useful information.
- We find that neither swing model is good, and district-level swing should be decreasing in x_i.

Figure: Local minus national swing versus original vote share in District 1: $p = 0.1$



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- For example, for the 2017 UK election in England, we can use the exact 2015 results, a national poll estimate, and a swing model to get a point estimate of the vote shares in each district.

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- This gives 100 different simulated elections each based on a realistic starting point.
- The method seems to work quite well (in this example it was more accurate than prediction markets and professional forecasters (ElectionsEtc site)).

Table: Forecast seats won by Conservative party using perfect national exit poll (100 simulated elections)

Nation	Real	Point	Minimum	Median	Maximum
England	296	299	280	295.5	309
Scotland	13	16	6	12	19
Wales	8	9	4	9	14

Table: Forecast seats won by Conservative party using pre-election opinion poll (100 simulated elections)

Nation	Real	Point	Minimum	Median	Maximum
England	296	308	295	312	326
Scotland	13	16	8	12	18
Wales	8	9	4	9	13

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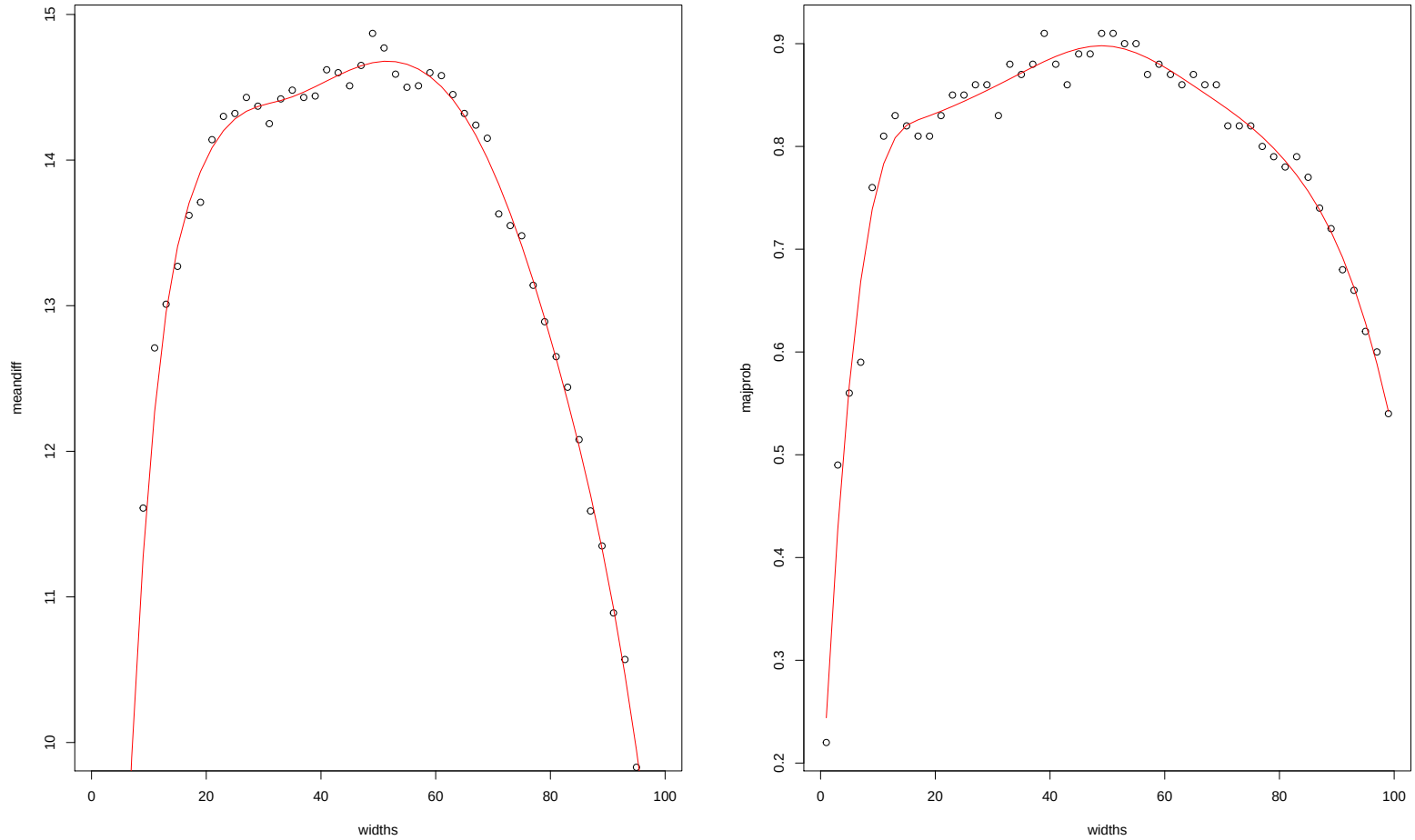
- Results are shown in Figure 3, which clearly indicates that neither extreme yields best results.

Figure: Campaign management simulations. Extra seats (L) and probability of majority (R), by width.



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- The districts need not be geographically based.
- Ask if you want to see the code — I aim to release it publicly "as time permits".