Swing Models

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Election time in Partisania

- The Definitely Scientific Party vies with the Recalcitrantly Anti-Intellectual Party using single-member plurality voting in districts. How many seats will they win in parliament?
- For decades, two competing estimation methods have been used.
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District	National	1	2	3	4	5
Election 2016	0.40	0.40	0.32	0.48	0.10	0.70
Election 2020 polling	0.60	?	?	?	?	?
Uniform	0.60	0.60	0.52	0.68	0.30	0.90
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Our answer: neither

- Uniform and proportional fail some natural axioms.
- We find a simple model that does satisfy all axioms:
  - defined piecewise for positive and negative swings;
  - of proportional type;
  - considers nonvoters rather than voters.
- Dataset of tens of thousands of US Congressional elections: piecewise model consistently performs a little better than others.
- Conclusion: throw out your old swing model, and replace with the new one. And keep looking for better models!
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Basic setup

- Assume K districts of equal size and two parties, A and B, contesting all districts.
- Unless otherwise specified we state results for party A, whose vote share is denoted x_i.
- The aggregate vote share is denoted $\overline{x}$.
- We consider two elections: one for which we know the results and another for which we don't. For the latter, we use prime for everything: $x'_i, \overline{x'}$.
- In addition to election prediction, this comes up in the study of gerrymandering and electoral system design, where we need to discuss counterfactuals.

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Definition

The district-level swing in district i is given by

$$s_i := x_i' - x_i.$$

The aggregate swing is given by

$$s := \overline{x}' - \overline{x}.$$

By a naive swing model we mean a prediction of x' of the form $s_i = f(x_i, s)$ where $f \equiv f_A$ is a fixed function (depending only on A but not i or s).

- Example: uniform swing, for which $s_i = s$.

Desirable properties of a swing model

- $$\frac{1}{K}\sum_{i=1}^{K} f(x_i, s) = s \qquad \textbf{(mean swing condition)}. \qquad (a1)$$

- $$0 \le x_i + f(x_i, s) \le 1 \qquad \textbf{(respecting bounds)}. \qquad (a2)$$

- $$f(x_i, s) + f(1 - x_i, -s) = 0 \qquad \textbf{(neutrality)}. \qquad (a3)$$

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Bad news

Proposition

No naive swing model can satisfy both (a1) *and* (a2)*.*

- We need a larger family, so we consider swing models of the form $s_i = f(x_i, s, \overline{x})$.
- Example: proportional swing, for which $s_i = sx_i/\overline{x}$.
- From the above example, this still fails to satisfy (a2).

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A new model

- Define
$$f(x_i, s, \overline{x}) = \begin{cases} s\frac{1-x_i}{1-\overline{x}} & \text{if } s \geq 0; \\ s\frac{x_i}{\overline{x}} & \text{if } s < 0. \end{cases}$$
- This looks like proportional swing but differs by considering positive and negative swings differently.
- Justification:
  - suppose in each district there are swing voters as well as partisans;
  - in districts where A already scores highly, there are relatively few swing voters left to convince;
  - in districts where A scored relatively low, there is more chance of winning over swing voters;
  - if the swing is away from A, the reverse is true (alternatively, the same is true of B).

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Table: Axioms satisfied by swing model

Model/Axiom	a1	a2	a3
uniform	✓	✗	✓
proportional	✓	✗	✗
truncated uniform	✗	✓	✓
linear in s	✓	✗	(✓)
piecewise	✓	✓	✓

Final slide

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Table: Results for swing models on standard dataset. Bold entries indicate the best performance among the models on the given measure corresponding to the column for the given dataset.

dataset	model / measure	winner	sign	bounds	mean-square
unc0.75	uniform	0.932	**0.497**	**1.000**	0.00747
unc0.75	proportional	**0.933**	**0.497**	0.999	0.00756
unc0.75	piecewise	0.930	**0.497**	**1.000**	**0.00728**
unc1.0	uniform	**0.904**	0.498	0.832	0.0381
unc1.0	proportional	0.904	0.539	0.884	0.0389
unc1.0	piecewise	0.892	**0.604**	**1.000**	**0.0360**
cont only	uniform	**0.855**	**0.678**	**1.000**	0.00521
cont only	proportional	0.853	**0.678**	0.999	0.00533
cont only	piecewise	0.852	**0.678**	**1.000**	**0.00509**