At ith step, swap $a[n+1-i]$ with $a[j]$, j uniformly randomly chosen from $1..n-i$. Claim: a now represents a uniform sample from the set of cyclic permutations of the original array.

- Examples: $j = n - i$ for each i yields $n \to n-1 \cdots \to 1 \to n$; $j = 1$ for each i yields $1 \to \cdots n \to 1$.
- Number of swaps is always $n - 1$. Other quantities of interest: number of moves by a given element, distance moved by a given element, total distance moved.

Proof of correctness

- $\mathcal{C}_n \cong \mathcal{C}_{n-1} \times \mathcal{N}$ via maps

$$(\sigma, q)^{\uparrow} = \sigma^* \tau \qquad \text{where } \tau \text{ is the transpositi}$$

$$\sigma_{\downarrow} = (\tau\sigma)_* \qquad \text{where } \tau \text{ is the transpositi}$$