Why does uniform swing work so well?

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Motivation

- The Definitely Caring Party vies with the Radical Anti-Intellectual Party in a state election using single-member plurality voting in districts.
- How many seats will they win in parliament? District-level polls are expensive, and we usually only have state-level information.
- What might happen if we change the district boundaries or use multi-member districts? What about potential demographic changes? Such electoral design questions also call for guesswork on district-level vote changes.
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General setup

- Assume K districts of equal size and two parties, A and B, contesting all districts under plurality voting.
- Unless otherwise specified we state results for party A, whose vote share in district i is denoted x_i.
- The aggregate vote share is denoted $\overline{x}$.
- We consider two elections: E for which we know the results and E' for which we don't. We use prime to label everything: $x'_i, \overline{x'}$.

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Before last year

- A model of inter-election swing is an estimate y_i of each x'_i (or alternatively of the district-level swing $x'_i - x_i$) given only x_i, $\overline{x'}$ and $\overline{x}$.
- The national swing $s := \overline{x'} - \overline{x}$ and this is typically the only information we have, or can estimate well, about E'.
- There are only two swing models in the political science literature and practice:
  - (uniform) $y_i = x_i + s$
  - (proportional) $y_i = x_i + sx_i/\overline{x} = x_i(1 + s/\overline{x}) = x_i\overline{x'}/\overline{x}$.

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Last year's talk

- Bernie Grofman and I presented three obvious axioms for swing models (respect means, respect bounds, neutrality).
- Uniform swing fails one axiom, proportional fails two.
- A modification of proportional swing (piecewise model) does satisfy all three. It has the form

$$y_i = \begin{cases} s\frac{1-x_i}{1-\overline{x}} = x_i\left(\frac{1-\overline{x'}}{1-\overline{x}}\right) + \frac{\overline{x'}-\overline{x}}{1-\overline{x}} & \text{if } s \geq 0; \\ x_i\left(1 + s/\overline{x}\right) & \text{if } s < 0. \end{cases}$$

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- We present several more conceptual reasons against uniform swing.
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Example: the models *may* be very different

- Uniform: same change in each district.
- Proportional: larger changes in already strong districts.
- Piecewise: smaller changes in already strong districts.

District	National	1	2	3	4	5
Election 1	0.40	0.40	0.32	0.48	0.10	0.70
Election 2 polling	0.60	?	?	?	?	?
Uniform	0.60	0.60	0.52	0.68	0.30	0.90
Proportional	0.60	0.60	0.48	0.72	0.15	1.05
Piecewise	0.60	0.60	0.547	0.653	0.40	0.80

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Reason against uniform swing #2: regression to the mean

- Suppose that x'_i is x_i plus random noise with mean s.
- The well-known "regression to the mean" effect shows that we expect the best linear fit to the data to have (for positive swings) a positive intercept and a slope that is positive but less than 1.
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More details

- Fitting $y = a + bx$ by ordinary least squares regression to the data, we find $\hat{a} = r_{xy} s_y / s_x$. For positive swings, $\hat{a}$ should be positive.
- Thus if the sample correlation is not perfect and the sample variances approximately the same (as they will be by assumption), then $\hat{a}$ is likely strictly less than 1.

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Mitigating reason: political processes

There are reasons why x'_i might move in the opposite way from regression to the mean.

- demographic reasons such as in- and out-migration make districts safer for the winning party;
- an incumbent of a given party may, especially if winning by large margins, discourage high quality challengers.

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How precise do our results need to be?

- For many purposes, we only care about statewide results: how many seats each party wins, or just which party has a majority.
- In such cases, we expect errors in seat-level predictions to average out across seats, if there are enough seats.
- The three models perform almost identically on on the over 34000 pairs of contested elections in the dataset.
- They "predict" the overall winner of the statewide election over 92% of the time and the mean absolute error in the seat fraction won by party A is about 5%.

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Table: Results for swing models on standard dataset, over 34000 district-level contested elections

dataset	model	winner	sign	bounds	ℓ_2^2	ℓ^∞	ρ
cont only	unif	0.874	0.676	1.000	0.005	0.509	0.890
cont only	prop	0.875	0.676	0.999	0.005	0.516	0.889
cont only	piece	0.874	0.676	1.000	0.005	0.504	0.890

Table: Results for swing models on standard dataset, contested elections with at least 30 districts. Fraction of times where 95% confidence interval from linear regression actually contains the model parameter.

dataset	model	slope	intercept
cont only	unif	0.676	0.685
cont only	prop	0.559	0.584
cont only	piece	0.736	0.747

Over all elections, the mean slope is around 0.9 and mean intercept 0.05.

Crudeness of results

Each here (AL 1998, CA 2000) satisfies the confidence interval criterion for uniform swing.

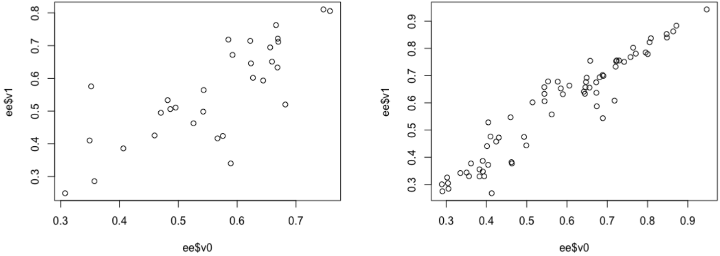

Table: Predictions for "competitive" example with 2 parties and 2 districts, swing of 2ε to A

	(A,1)	(A,2)
original	$1/2 - \varepsilon$	$1/2 + \varepsilon$
uniform	$1/2 + \varepsilon$	$1/2 + 3\varepsilon$
proportional	$1/2 + \varepsilon - 4\varepsilon^2$	$1/2 + 3\varepsilon + 4\varepsilon^2$
piecewise	$1/2 + \varepsilon + 4\varepsilon^2$	$1/2 + 3\varepsilon - 4\varepsilon^2$

So it is not surprising that in many real elections, all methods have fairly similar performance.

Conclusions

- The uniform swing model has more negative features than previously discussed.
- It works OK for many purposes on real vote data, but so do the other models.
- The answer to the question in the title of the talk is: "because we measure crudely, because of cancellation across districts, and because of cancellation caused by competing political processes - but the piecewise model dominates it."
- The piecewise model also deals better with extreme cases and (I guess) non-electoral applications.
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Addendum - another application

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- Exam scaling is one possible application.
- I am not sure about the need for the third axiom in this case, but the piecewise model is the only one we have looked at that satisfies the first two.
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