Decisiveness, power, values

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CMSS Seminar, 2011-05-10

Key references

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Key motivating examples of simple games

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- Disequilibrium games: for a given noncooperative game and fixed profile of actions, declare a subset to be winning if is a witness to the profile not being a strong Nash equilibrium. Examples: voting rules with the sincere profile.

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Basic concepts of TU games and simple games

monotonicity $S \subseteq T \implies v(S) \leq v(T)$.

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proper simple game $S \in W, T \in W \implies S \cap T \neq \emptyset$.

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- Treaty of Nice (currently in force) uses weights (totalling 345) but has more conditions. A coalition is winning iff it has at least 50% of the countries, 74% of the weights, 62% of the population.
- Treaty of Lisbon (from 2014): coalition wins iff it has at least 55% of countries and 65% of population. This method is easily implemented if new members join, and avoids complex negotiations over weights.

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Dummy	$\xi_i(G) = 0$ if i is a dummy in G.

The Shapley value

- Shapley proved that there is a unique efficient value satisfying Anonymity, Dummy and Linearity. Explicitly it is given by

$$\sigma_i(G) = \frac{1}{n!} \sum_{S \subseteq X} (n - |S|)!(|S| - 1)! \left[v(S) - v(S \setminus \{i\}) \right].$$

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- The idea is to consider all possible orders of players with equal probability, and give player i its expected marginal contribution.

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$$\xi_i(G) = \sum_{k=0}^{n} p(n,k) \sum_{|S|=k, S\subseteq X} [v(S) - v(S \setminus \{i\})]$$

where $p(n,k) \geq 0$ and the following identities hold

$$\sum_k \binom{n-1}{k-1} p(n,k) = 1$$

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- If all $p(n,k) \neq 0$, the semivalue is called regular.

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- Regular semivalues satisfy Young sensibility: if the marginal contribution to each S is strictly higher in one game than another, then the ξ_i have the same relation.
- The class of probabilistic values is even more general - the coefficients p can depend on S and not just on $|S|$.

Some explicit semivalues

- (binomial (for fixed $p \in [0,1]$))

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$$\Phi(G) - \Phi(G_{-\{i\}}) = \xi_i(G)$$
for all $G = (X, v) \in \mathcal{G}$ such that $X \neq \emptyset$. Here $G_{-\{i\}}$ is the game with player set $X \setminus \{i\}$ and the same v.

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- The initial condition $\Phi(\emptyset, v) = 0$ is usually assumed.
- There is a unique efficient value having a potential function, and it is the Shapley value. Explicitly, the potential looks like
$$\Phi(G) = \sum_k \frac{1}{k\binom{n}{k}} \sum_{|S|=k, S\subseteq X} v(S).$$

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- The answer: ξ has a potential if and only if it satisfies Myerson's balanced contributions axiom:

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- In particular, every semivalue has a potential function. Explicitly:

$$\Phi(G) = \sum_k p(n,k) \sum_{|S|=k} v(S)$$

the expected value of a coalition chosen randomly according to the weights $p(n,k)$.

Measuring power in simple games

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- The underlying idea is to measure the extent to which a player is important for winning coalitions. A key observation is that for weighted majority games, the relative power of the players can vary dramatically from the relative weights.
- Much has been written, but no standard definition of a power measure/index has been agreed. There are many conceptual confusions in the literature and some controversy.

Concepts of power in simple games

- Felsenthal and Machover: there are at least two kinds of "power" and previous authors have conflated them. P-power deals with distribution of the spoils of power; I-power deals with decisiveness. The former may not be well-defined, but the latter is. The former is always relative, but the latter is absolute.

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- Laruelle and Valenciano: there are at least two kinds of situation, and previous authors have conflated them. Take it or leave it committees must only vote; pure bargaining committees involve complex negotiations. In the first case, decisiveness is not as important as "success". P-power in fact is related to decisiveness via bargaining committees.

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- The most common measure is the Banzhaf measure, the specialization of the Banzhaf semivalue: the probability that i is decisive for a uniformly randomly chosen coalition containing i.
- This measure was discovered first for simple games by Penrose and rediscovered by Banzhaf (1965) in the context of a court case over the Nassau County Board of Supervisors (weighted voting game $[16; 9, 9, 7, 3, 1, 1]$).

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- Example: EUCM under Treaty of Nice has $C \approx 0.02$. Very hard to pass any motion, hence the need for reform in Treaty of Lisbon (which currently has $C \approx 0.13$).

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- Idea: define an individual decisiveness measure to be the restriction of a semivalue to $\mathcal{SG}$. Note: such functions satisfy Anonymity, Positivity, Dummy, and Modularity (the replacement for Linearity).

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  - Coleman index: $C = 0.492$.

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- Efficiency is an obvious requirement if a fixed prize is being divided, and this usually leads to the Shapley value (Shapley-Shubik index). Otherwise efficiency is meaningless and should be dropped.
- Collective decisiveness certainly is important, so individual power measures do measure something important, even if it is not "power".

Manipulation and query model

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- Let $\overline{Q}$ be the expected number of queries required. Then $n + 1 - \overline{Q}$ is essentially the potential of the Shapley value of the manipulation game.
- If each voter can have a different cost to recruit (as in bribery), a TU (cost) game is more appropriate than a simple game, but similar ideas should work.

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- This is a substantial advance in the theory of measures of manipulability. There has been no definition of what such a measure should be, and no desirable axioms listed. Previous measures have been rather crude.
- Some (not all) of the previously used measures can be interpreted as semivalues, but not always regular ones. Our new approach allows a principled choice of measure for a given situation.

Manipulability example

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- Assuming risk-neutral voters, there are two types of minimal winning coalitions: a single cba (respectively bca) votes for b (respectively c). Note that this game is not strong.
- The winning coalitions are those containing at least one bca or cba. The Coleman index is $15/16$. The relative Banzhaf (or Shapley-Shubik) index of each cba or bca is $1/4$, and abc voters are dummies.