

# What is Computational Social Choice?

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[www.cs.auckland.ac.nz/~mcw/blog/](http://www.cs.auckland.ac.nz/~mcw/blog/)

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References

Computational microeconomics

Social choice

Game theory and mechanism design

Social choice mechanisms

# Centre for Mathematical Social Sciences

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- ▶ <http://cmss.auckland.ac.nz>.

## Good survey articles

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- ▶ Commercial problems have dominated research on the CS side, but a shift toward a broader viewpoint is evident.
- ▶ No official name: “computational (micro)economics”, “algorithmic game theory”, “algorithmic mechanism design”?



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  - ▶ They cooperate/compete by playing a strategic game.

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- ▶ Electronic voting?



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- ▶ ACM Conference on Electronic Commerce, Symposium on Algorithmic Game Theory, Workshop on Computational Social Choice
- ▶ Papers: The Complexity of Computing Nash Equilibria, Selfish Routing and the Price of Anarchy, Approximate Mechanism Design without Money, Truthful Fair Division, Combinatorial Auctions

## Contributions flow both ways

- ▶ Econ  $\rightarrow$  CS: distributed computing and networking protocols (such as TCP-IP) have traditionally assumed that components cooperate. However incentives and selfish preferences cannot be ignored. Rational behaviour can lead to suboptimal outcomes if not controlled.

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- ▶ CS  $\rightarrow$  Econ: traditional models use mathematical existence results such as fixed point theorems. However computational and communication complexity cannot be ignored. Strategies and solutions may not be practically computable.

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- ▶ Used for millenia in human political decision-making (voting, elections, planning, where to build an airport, allocation of objects to people, ...).
- ▶ Very often we require only a single winner (**social choice function**), and tiebreaking procedures are almost always needed. Randomized tiebreaking leads to objects that are not strictly speaking social choice functions.



## Some social choice functions

- ▶ Scoring rules: fix a vector  $1 = w_1 \geq w_2 \geq \dots \geq w_m = 0$ . Voter awards  $w_1$  points to its most preferred alternative,  $w_2$  to second, etc. Highest total score wins. Famous examples: plurality ( $w_i = 0$  for  $i > 1$ ); Borda (weights are equally spaced); veto ( $w_i = 1$  for  $i < m$ ).

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- ▶ Condorcet rules: if the majority relation has a clear winner, choose it. Otherwise choose something else. Example: Copeland rule: award  $\pm 1$  for each pairwise majority victory/defeat, highest total wins.
- ▶ Dictatorship: one voter decides the result, irrespective of the preferences of others.

## Classic paradoxes of social choice theory

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- ▶ Participation: the winner may not remain the winner when extra voters rank it first.

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## (Noncooperative) game theory

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- ▶ Classic examples: Chicken, Battle of the Sexes, Prisoners' Dilemma. Suboptimal outcomes can occur because of misalignment of individual incentives, but sometimes don't. It depends on the structure of the game.

## Example: load balancing

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- ▶ Each of these strategy profiles is a **Nash equilibrium**: given that all other players play the strategy, no player has incentive to deviate. However it is not a dominant strategy equilibrium: if some players deviate, sticking with the strategy may be bad.

## Mechanism design

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- ▶ Each other player has private utility information called its **type**  $\theta$ , and must report some type  $\hat{\theta}$ . Let  $\Theta$  be the profile of all players types. If designer knew  $\Theta$  or players always report  $\Theta$ , the job is easy. However, players can strategically lie,  $\hat{\Theta} \neq \Theta$ .

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- ▶ The designer announces an allocation rule  $R_2$  (including transfer payments), and uses this on the reported types. Designer aims for  $R_2(\hat{\Theta}) = R_1(\Theta)$ .

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- ▶ Classic nonexample: first-price auction. The winner pays its own bid.
- ▶ Important nonexample: (later) nondictatorial social choice functions.

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- ▶ If we announce this then players have an incentive to bid lower than  $v_i$  (how much depends on their perception of the bids of other players - the game is complicated).
- ▶ However, if we announce  $R_2$ : “give the object to the highest bidder, and charge him the second-highest bid”, there is no incentive to bid untruthfully and players may as well report  $v_i$ .

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- ▶ If players are truthful, standard shortest path algorithms will optimize social welfare (minimize total cost). However, they have clear incentive to report a higher cost than they actually incur.
- ▶ The general **Vickrey-Clarke-Groves mechanism** yields a nice solution. We pay  $e$  zero if  $e$  is not in the cheapest path, and otherwise pay its reported cost plus a “bonus” equal to its “contribution”: the increase in cost of the cheapest path if  $e$  were deleted.

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- ▶ VCG only works when we want to maximize the total utility of the players, not for other measures of welfare.
- ▶ There is much research on how to get around these difficulties using approximations.

## The CS contribution

- ▶ Computational complexity: mechanisms may be arbitrarily complex. Strategies, equilibria, ... may be NP-hard (or worse) to compute. In fact they often are.

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- ▶ Approximation algorithms: the standard response to hard optimization problems. Concepts such as approximation ratio.
- ▶ Worst-case (non-Bayesian) analysis.

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- ▶ The main problem is that the outcome of the game is easily predicted only when there is a unique dominant strategy (truthtelling) for all players.
- ▶ In general there will be many reasonable “predictions” (usually these are Nash equilibria). Problems: in the worst case Nash equilibria are likely not computable in polynomial time [DGP2009]; there are far too many of them.
- ▶ Which equilibrium do we look at in order to measure the overall welfare? This leads to ideas such as **price of anarchy**.

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- ▶ The strategic action of each voter is to report a preference order (possibly untruthful).
- ▶ There are no payments.
- ▶ The outcome is a single alternative and this determines the allocation rule (each player receives some “payoff” from that alternative winning).



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- ▶ The main problem is that in this model we have no way of measuring utility, or of comparing utilities between players. Money is a convenient way of getting past this problem, which is why interesting truthful mechanisms can exist in commercial settings.
- ▶ Manipulation by coalitions is sometimes possible where individual manipulation is not.

## Coalitional manipulation example

- ▶ Consider a voting situation with 3 alternatives  $a, b, c$  and having 4  $abc$ , 3  $bca$  and 2  $cab$  voters. Under the plurality rule, the sincere winner is  $a$ .

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- ▶ However, if the  $bca$  voters all vote strategically as  $cba$ , then  $c$  wins.
- ▶ This is an example of a mechanism that is individually truthful, but not jointly - a group has an incentive to deviate. Voting sincerely is a Nash equilibrium, but not a strong Nash equilibrium.



## Computational response to Gibbard-Satterthwaite

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- ▶ Successes: Instant Runoff Voting is NP-hard to manipulate by a single voter [BO1991]; weighted voting rules are almost always NP-hard to manipulate by a coalition, even for a fixed number of alternatives [CSL2007].
- ▶ Problems: NP-hardness is only a worst-case guarantee. Most rules seem easy to manipulate in practice (based on simulation and some analytic results, e.g. [RPW2010]).

## Some topics of current interest to me

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- ▶ Asymptotic probabilistic measures of manipulability (Geoffrey Pritchard)
- ▶ Implementation of social choice rules using different solution concepts.



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- ▶ A (pure strategy) Nash equilibrium is always reached in  $O(m^2n^2)$  iterations.
- ▶ Small changes to hypotheses lead to a failure to coordinate.
- ▶ Above results are [MPJR2010]. What happens for other voting rules?

## Convergence via polling

- ▶ Consider the previous model, but each agent has **inertia**, a new measure of its risk attitude and available information. Also, instead of sequentially, agents vote simultaneously, and they repeat this procedure. Can interpret as a sequence of opinion polls, and agents strategize based on the incomplete information gleaned from polls.

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- ▶ For some inertia distributions, convergence to an equilibrium where only two candidates get votes (**Duverger's law**). For others, no convergence.
- ▶ In the zero inertia case, announcing Plurality leads to Instant Runoff.
- ▶ Idea of Reyhaneh Reyhani (PhD student), explored in her thesis work.

## Safe manipulation

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- ▶ S & W proved an analogue of Gibbard-Satterthwaite, so we can't avoid safe manipulation.
- ▶ Can complexity help? Can a safe manipulation be found in polynomial time? Egor Ivanovskii (CS380 project) has solved this open problem for the Borda rule.